Analysis of Superpressure Balloon using the ideal gas law.

$$P_{super} = P_{gas} - P_{air} \qquad (1)$$

We can write the ideal gas equation for the gas inside the balloon:

$$P_{gas}V = \frac{m_{gas}}{M_{gas}} RT_{gas} \qquad (2)$$

Since the system is floating, we know the mass of the air displaced is m_{system}. So we can also write the idea gas equation for the air displaced by the balloon.

$$P_{air}V = \frac{m_{system}}{M_{air}} RT_{air} \qquad (3)$$

We assume the volumes are equal, so we can subsitiute one into the other.

$$P_{gas} = P_{air} \left[\frac{\frac{m_{gas}}{M_{gas}} RT_{gas}}{\frac{m_{system}}{M_{air}} RT_{air}} \right] \qquad (4)$$

Re-arrange and cancel R:

$$P_{gas} = P_{air} \left[\frac{m_{gas} T_{gas} M_{air}}{M_{gas} T_{air} m_{system}} \right] \qquad (5)$$

Now we can use the definition of superpressure (1):

$$P_{super} = P_{gas} - P_{air} \qquad (1)$$

$$P_{super} = P_{air} \left[\frac{m_{gas} T_{gas} M_{air}}{M_{gas} T_{air} m_{system}} - 1 \right] \qquad (6)$$

Substituting in our expression for P_{air}:

$$P_{air} = \frac{m_{system} RT_{air}}{M_{air} V} \qquad (3)$$

$$P_{super} = \frac{m_{system} RT_{air}}{M_{air} V} \left[\frac{m_{gas} T_{gas} M_{air}}{M_{gas} T_{air} m_{system}} - 1 \right] \qquad (7)$$

$$P_{super} = \frac{R}{V} \left[\frac{m_{gas}}{M_{gas}} T_{gas} - \frac{m_{system}}{M_{system}} T_{air} \right] \qquad (8)$$

We define supertemperature in the same way as superpressure:

$$T_{super} = T_{gas} - T_{air} \qquad (9)$$

$$P_{super} = \frac{R}{V} \left[\left(\frac{m_{gas}}{M_{gas}} - \frac{m_{system}}{M_{air}} \right) T_{air} + \frac{m_{gas}}{M_{gas}} T_{super} \right] \qquad (10)$$

We can reasonably say the superpressure due to the temperature dominates, so

$$\frac{P_{super}}{T_{super}} \approx \frac{m_{gas}}{M_{gas}} \frac{R}{V} \qquad (11)$$