Submarine Simulation Design

The goal of this exercise is to learn to implement a physics model in a Trick simulation in a clear and organized way.

> *"The computing scientist's main challenge is not to get confused by the complexities of his own making."*
> *— Edsger W. Dijkstra*

Project: Submarine Simulation

Using Newton's Second Law, and only the forces of gravity, buoyancy and hydrodynamic drag, described in the following pages, complete a Trick based simulation which continuously computes the position and velocity of a submarine.

Simplifications

The submarine will only move in one-dimension, that is up or down.

In this simulation, we'll assume that there is no loss of energy due to cooling of ballast air.

Control

In this simulation, we'll control the position of the submarine by pumping air into or out of a ballast tank, which in turn displaces water.

General Strategy for Developing a Dynamic Simulation

“Dynamic” means “characterized by change”.

In a dynamic simulation things like position, and velocity, continuously change over time. Lots of other things might change as well depending on what you are simulating. The amount that something changes per unit of time is its **rate of change,** or **time derivative.**

For example:

- Acceleration is a rate of velocity change .
- Velocity is a rate of position change.
- Current is a rate of change of charge.

If we know the rate of change of something, then we can add up all the little changes over time to keep track of its value. This “adding up the little pieces” is called “numerical integration”.

This is the basic strategy for designing Trick dynamic simulations. We calculate rates, and then we numerically integrate them.

General Strategy for Motion

To determine the motion of an object we generally start with Newton's Second Law:

$$F = ma$$

That is, force equals mass times acceleration.

This allows us to determine the rates that effect motion, that is: acceleration and velocity.

Solving for acceleration, we get :

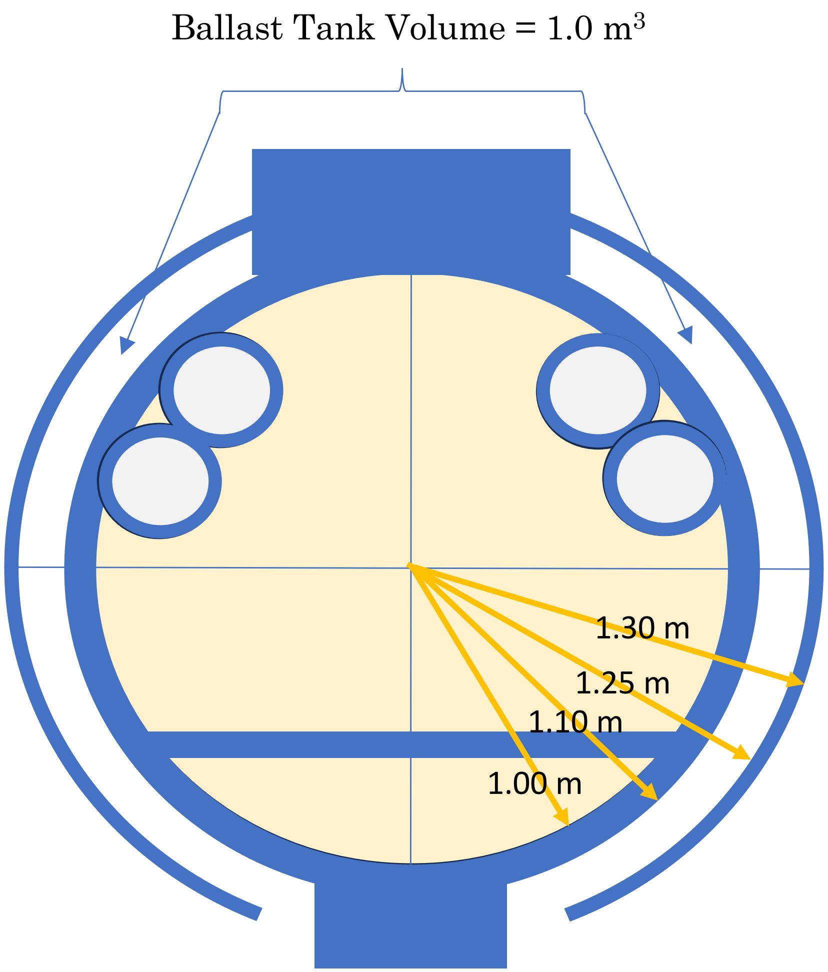
$$a = F/m$$

This is the form of Newton's Law that we generally use.

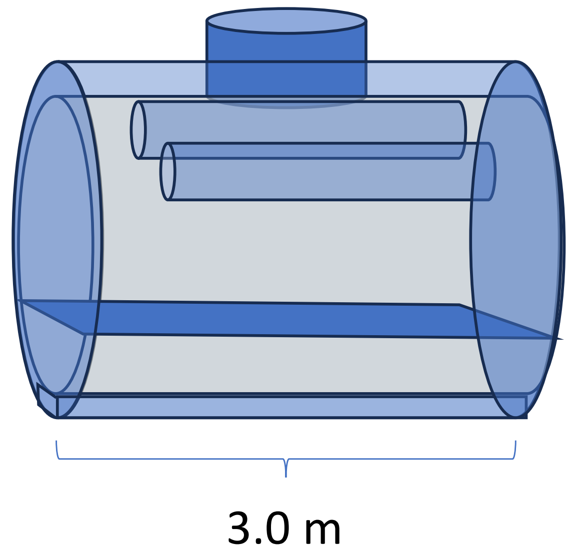
So, if we have a force acting on a mass, we can determine its acceleration. Then, we can then numerically integrate that acceleration to get velocity and then integrate the velocity to get position.

In our simulation our rates will be acceleration, velocity, and power. For each time step ($\boldsymbol{\delta}$t) of our simulation, we'll integrate our rates over the time period t $\rightarrow$ t + $\boldsymbol{\delta}$t to get the next velocity, position, and energy state. A Trick integration job will calculate this for us.

Specifications of Our Submarine



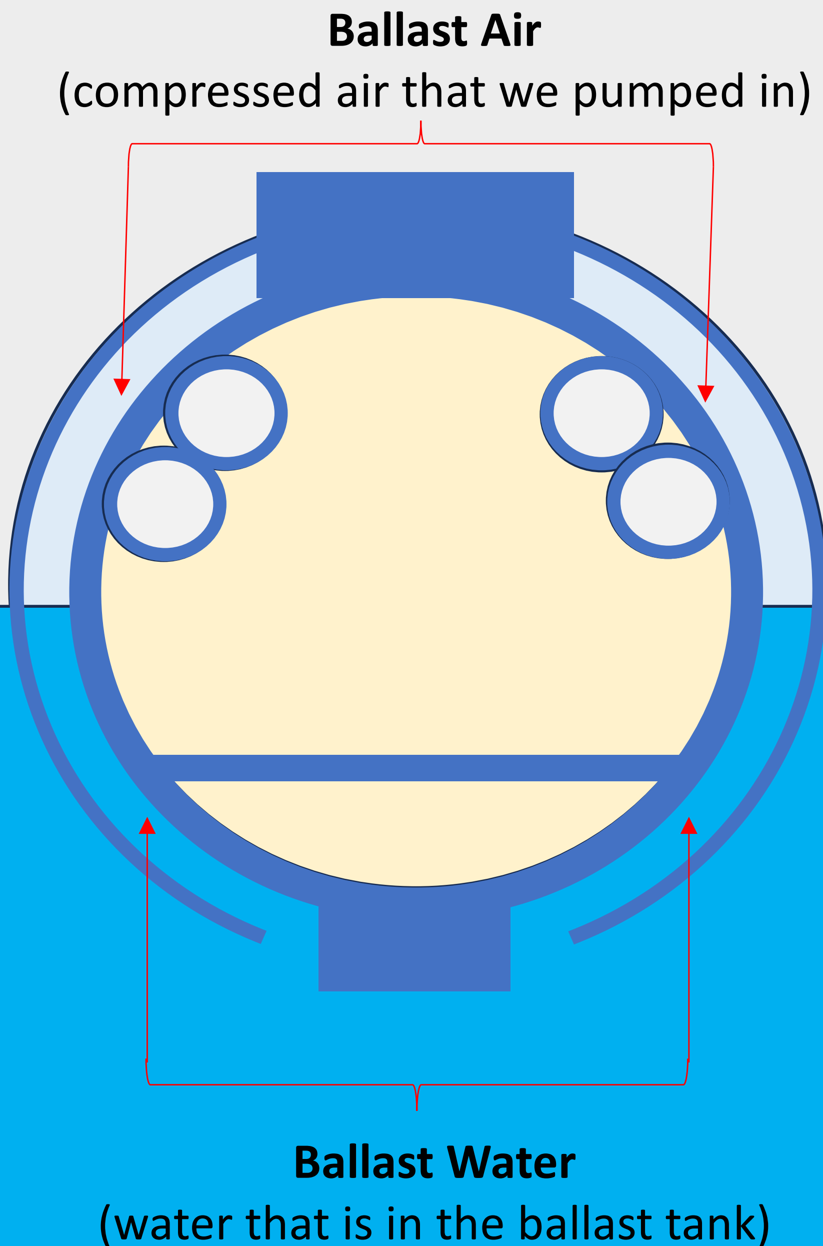
Mass of Empty Submarine
10000.0 kilograms



Coefficient of Drag (C_d) = 0.5

You're right! It's an ugly submarine,
but it simplifies the simulation modeling.

More Terms



NOTE that the *ballast air volume* plus, the *ballast water volume* is the *ballast tank volume*.

A Bit About the Ideal Gas Law

$$PV = nRT = \text{Energy in the system (E)}$$

If we compress air in a container, its volume will go down, and its pressure will go up.

If we then decompress the air, its volume will go up and, its pressure will go down.

If the system doesn't lose heat, which we will assume for simplicity, the energy (E) will remain the same. So, $P = E/V$ and $V = E/P$.

If we have a fixed amount of air in the ballast tank, its volume will decrease as pressure increases. What doesn't change is the product of P and V, that is the energy (E). So, we will control the buoyancy of the submarine by pumping <u>energy</u> into the system, which we'll call ballast energy. Then we can calculate the volume from: $V = E/P$.

Acceleration

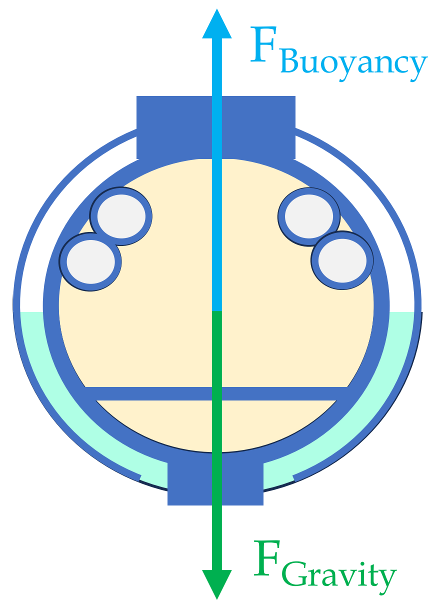
To calculate acceleration, we'll use Newton's Second Law as usual. The force is the total force acting on the submarine.

(1) $$\vec{a} = \frac{\vec{F_{total}}}{m}$$

The total force on our submarine is the sum of:

- the force of gravity
- the force of buoyancy
- the drag force produced by the submarine pushing through the water.

(2) $$\vec{F_{total}} = \vec{F_{gravity}} + \vec{F_{buoyancy}} + \vec{F_{drag}}$$



Notice that force is a vector. That is, it has a <u>magnitude</u>, and a <u>direction</u>.

If we affix a "Body" coordinate system to the submarine as shown, then the gravity force will act in the -y direction, and the buoyancy force in the +y direction. The drag force points the opposite direction of motion.

Gravity Force

Applying the acceleration of gravity to Newton's 2nd Law:

(3) $$F_{gravity} = m_{total} \cdot g$$

At sea-level, ***g*** is around 9.81 m/s^2. Since our submarine will at most descend a few miles, we'll treat *g* as a constant.

What determines the mass of the submarine?

Submarine Mass

(4) $$m_{total} = m_{fixed} + m_{hard\_ballast} + m_{ballast\_water}$$

hull	10000.0 kg
payload (passengers, etc.)	1500.0 kg
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m_{fixed}	11500.0 kg

$m_{hard\_ballast}$	0.0 kg

What's the mass of the ballast water in the submarine?

Ballast Water Mass

The mass of the water in the ballast tank is the volume of the water in the ballast tank times the density of water.

(5) $m_{ballast\_water} = V_{ballast\_water} \times \boldsymbol{\rho}_{\mathbf{water}}$

The volume of the water in the ballast tank is the volume of the ballast tank minus the volume filled with air.

(6) $V_{ballast\_water} = \mathbf{V}_{\mathbf{ballast\_tank}} - V_{ballast\_air}$

We also need to calculate the ratio of the ballast air to ballast tank volume for our GUI, to draw the water and air in the ballast tank.

(6.a) $ballast\_air\_ratio = V_{ballast\_air} \,/\, \mathbf{V}_{\mathbf{ballast\_tank}}$

Ballast Air Volume

(7) $V_{ballast\_air} = E_{ballast} / P_{water}$

Note that we must limit the volume of the ballast air to that of the ballast tank. That is $V_{ballast\_air}$ must be limited to less than or equal to $V_{ballast\_tank}$

Also, when we limit $V_{ballast\_air}$ we must recalculate $E_{ballast}$

What's the water pressure (P_{water}) at our submarine's depth?

Water Pressure

(8) $$P_{water} = \boldsymbol{\rho}_{\mathrm{water}} \times g \times depth + P_{atmosphere}$$

- $\boldsymbol{\rho}_{\mathrm{water}}$ is the density of water.
- g is the acceleration of gravity.
- $depth$ is our submarine's depth below the surface.

Note that this is the negative of our submarines y-position.

- $P_{atmosphere}$ is the atmospheric pressure at the surface of the water.

Drag Force

As the submarine moves upward, or downward, it will be subject to an drag force. Drag is a function of the submarine's shape [presented by the coefficient of drag (C_d)], the density of the water (***ρ***), and the cross-sectional area (A) perpendicular to the velocity (v). The drag force points in the opposite direction as the velocity.

(9)

$$F_{drag} = - \tfrac{1}{2} \boldsymbol{\rho} \, v \, | v | \, C_d \, A$$

For our submarine, we'll assume $C_d = 0.5$, the coefficient of drag for a sphere.

ρ (Density of Salt Water) = 1023.6 kg/m^3

We'll calculate the cross-sectional area from the outer radius of the outer hull of our submarine (Eq#).

(10)

$$A = outer\_hull\_outer\_radius \times 2 \times hull\_length$$

Force of Buoyancy

Buoyancy is a force on an object, that opposes gravity, by a fluid within which it's immersed. This force is equal to the mass of the displaced fluid times the acceleration of gravity.

(11) $$\boxed{F_{bouyancy} = m_{displaced\_water} \cdot g}$$

$$m_{displaced\_water} = (V_{hull\_disp} + V_{ballast\_air}) \cdot \boldsymbol{\rho}_{water}$$

- V_{hull_disp} is the volume of water displaced by the hull.

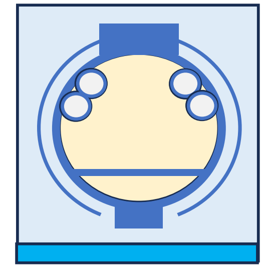
- $V_{ballast\_air}$ is the volume of air in the ballast tank. Note that this air <u>displaces</u> water from the ballast tank. This was calculated in equation #7.

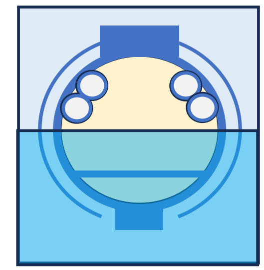
- $\boldsymbol{\rho}_{water}$ is the density of water.

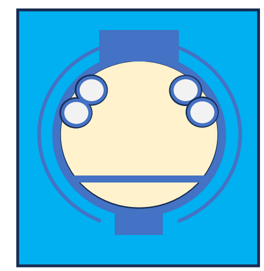
Hull Displacement Volume

(12)

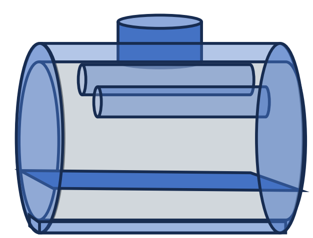
$$V_{hull\_disp} = \begin{cases} 0 & \text{if hull is not submerged.} \\ V_{hull} / (1 + e^{(-5.5\ depth)}) & \text{if hull is partially submerged, We can estimate the displacement.} \\ V_{hull} & \text{if hull is fully submerged} \end{cases}$$







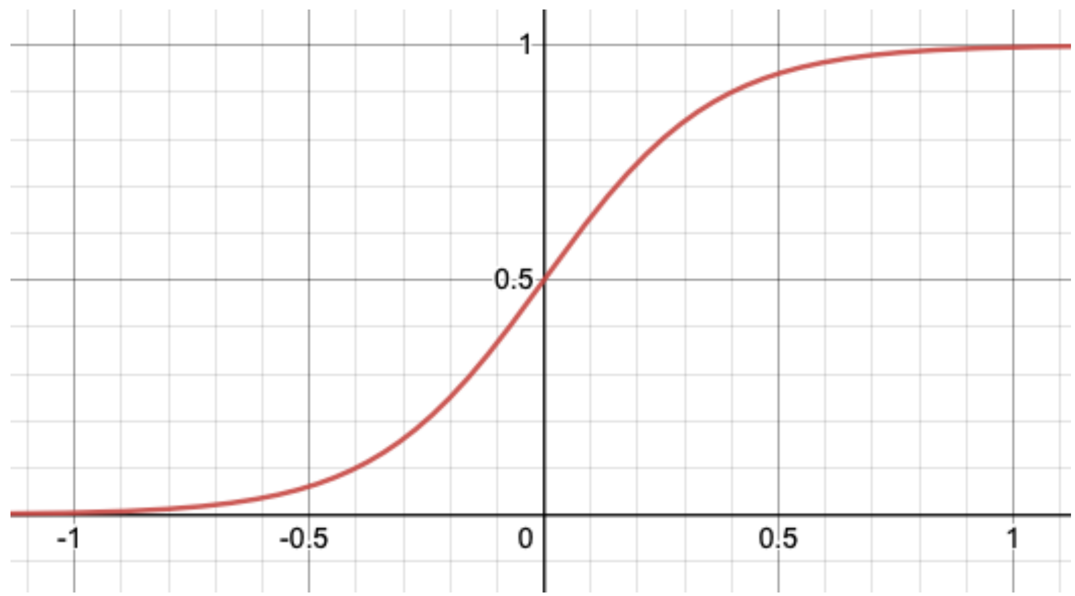
(13) $V_{hull} = \boldsymbol{\pi} \times (\text{inner\_hull\_outer\_radius})^2 \times \text{hull\_length}$



Side-Note About the Partially Submerged Hull Estimate

In the special case where the submarine is only partially submerged, we estimate the volume of water displaced by the hull using a sigmoid (S-shaped) function.

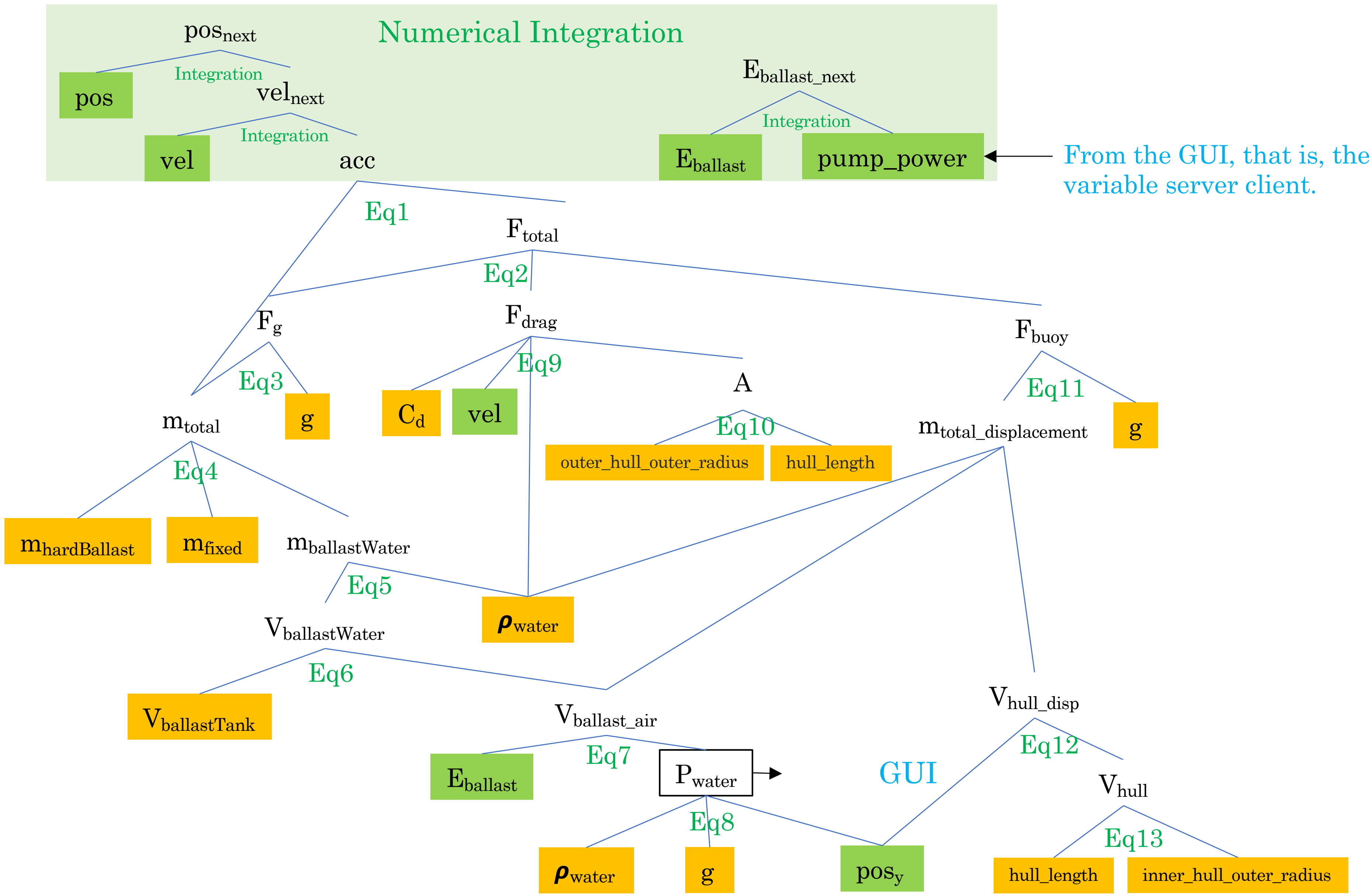
$$y = \frac{1}{1 + e^{-5.5x}}$$



Organizing Things

We now have the equations needed to calculate our acceleration, forces, masses, volumes, densities, so forth. On the next few pages, we'll look at our variables in terms of computational dependencies. This exercise can give us some valuable insight as to how we organize our simulation.

Computational Dependencies



Submarine Simulation State

What are the members of the struct or class that describes our submarine?

In the dependencies tree, the variables that don't have dependencies (in the yellow boxes), completely define the **parameters** of our submarine. The variables derived by numerical integration (in the green boxes) define the **dynamic state** of the submarine. Let's start with these.

Whereas pos and vel are 2-D arrays we're only going to update the vertical element (y) for now.

Finally, we need some control variables for communicating with our variable server client.

```
class Submarine {
    public:

    // Parameters
    double outer_hull_outer_radius;
    double inner_hull_outer_radius;
    double hull_length;
    double ballast_tank_volume;
    double hard_ballast_mass;
    double hull_mass;
    double payload_mass;
    double Cd;

    // State Variables (Uncalculated Variables)
       double pos[2];
       double vel[2];
    double ballast_energy;  // joules
    double pump_power;      // watts

    // Calculated Variables
       double acc[2];

    // Control Variable
    double ballast_air_ratio;     // To the GUI.
    double water_pressure_pascals;
    double water_pressure_PSI;    // To the GUI.
    int    pump_power_command;    // From the GUI.
```

Default Values

- Position of the submarine : (0, 0) m.
- Velocity of the submarine : (0, 0) m/s.
- Fixed mass components of the submarine
  - hull_mass = 10000.0 kilograms
  - hard_ballast_mass= 0.0 kilograms
  - payload_mass = 1500.0 kilograms
- Dimensions of the submarine
  - outer_hull_outer_radius = 1.30 meters
  - outer_hull_inner_radius = 1.25 meters
  - inner_hull_outer_radius = 1.10 meters
  - inner_hull_inner_radius = 1.00 meters
  - hull_length = 3.0 meters
- ballast_tank_volume = 1.0 cubic meters
- The coefficient of drag (C_d) : 0.5.

Density of Salt Water ($\boldsymbol{\rho}$) = 1023.6 kg/m^3
Acceleration of gravity (g) = 9.80665 m/s^2
Atmospheric pressure = ($P_{atmosphere)}$ = 101353.0 pascals = 14.7 $lbs/inch^2$