Stand-Alone Integration Library

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Introduction

The Stand-Alone Integration Library can be used within a Trick simulation, or independent of it.

Some examples of using these integrators can be found in the **examples/** directory.

- CannonBall uses the RK2Integrator.
- MassSpringDamper uses the EulerCromerIntegrator.
- Orbit uses the EulerCromerIntegrator.
- DoubleIntegral shows an example of a double integral.

class Integrator

Description

This base-class represents a numerical **integrator**.

Data Members

Member	Type	Access	Description
X_in	`double`	Protected	Independent variable value of the input state.
X_out	`double`	Protected	Independent variable value of the output state.
default_h	`double`	Protected	Default integration step-size
user_data	`void*`	Protected	A pointer to user defined data that will be passed to user-defined functions when called by the Integrator.

Constructor

```
Integrator(double h, void* udata);
```

Parameter	Type	Description
h	`double`	Default integration step-size
udata	`void*`	A pointer to user defined data that will be passed to user-defined functions when called by the Integrator.

Destructor

```
virtual ~Integrator() {}
```

Public Member Functions

```
virtual void step()
```

Derived classes should override this method to perform a numeric integration step, and then call `advanceIndyVar()` to advance the independent variable. The default behavior of this member-function is to call `advanceIndyVar()`.

```
virtual void load()
```

Derived classes should override this method to load/prepare the integrator for the next integration step. The default behavior is to set the input value of the independent variable to its previous output value, i.e, `X_in = X_out`.

`virtual void unload()`

Derived classes should override this method to disseminate the values of the output state to their respective destinations. The default behavior is to do nothing.

`virtual void integrate()`

Call `load()`, `step()`, and `unload()` in order.

`virtual double undo_integrate()`

Derived classes should override this member function to **undo** the effect of `integrate()` and return that last step-size. The behavior of this function is to set the output value of the independent variable to its previous input value, i.e, `X_out = X_in`, and then return the default step-size `default_h`.

`double getIndyVar()`

Returns the value of the independent variable (i.e, the variable over which you are integrating) If you are integrating over time, this value will be the accumulated time.

`double setIndyVar( double t)`

Sets the value of the independent variable. (i.e, the variable over which you are integrating) If you are integrating over time, this will be the accumulated time.

Protected Member Functions

`void advanceIndyVar()`

This member function advances the independent variable by the default integration step-size.

typedef DerivsFunc

Description

This typedef defines a type of C/C++ function whose purpose is to populate a state derivative array.

```
typedef void (*DerivsFunc)( double x, double state[], double derivs[], void* udata);
```

where:

Parameter	Type	Direction	Description
x	`double`	IN	Independent variable.
state	`double*`	IN	Array of state variable values.
derivs	`double*`	OUT	Array into which derivatives are to be returned.
udata	`void*`	IN	Pointer to user_data.

Example

```
void my_derivs( double t, double state[], double deriv[], void* udata) { ... }
```

class FirstOrderODEIntegrator

Derived from `Integrator` .

Description

This class represents an integrator for a first order Ordinary Differential Equation.

Data Members

Those inherited from Integrator plus:

Member	Type	Access	Description
state_size	`unsigned int`	Protected	Size of the state vector.
inState	`double*`	Protected	Input state vector to the integrator.
outState	`double*`	Protected	Output state vector from the integrator.
inVars	`double**`	Protected	Array of pointers to the variables from which input state vector values are copied.
outVars	`double**`	Protected	Array of pointers to the variables to which output state vector values are copied.
derivs_func	`DerivsFunc`	Protected	Function thats generates the function (an array of state derivatives) to be integrated.

This class introduces:

- Input and output state arrays.
- A function that calculates state-derivatives for the integration algorithm.
- Array of pointers to variables from which to load the input state array, and array of pointers to variables to which to unload the output state array.

Constructor

```
FirstOrderODEIntegrator( double h,
                         int N,
                         double* in_vars[],
                         double* out_vars[],
                         DerivsFunc func,
                         void* user_data);
```

where:

Parameter	Type	Description
h	`double`	Default integration step-size.
N	`int`	Number of state variables to be integrated
in_vars	`double*`	Array of pointers to the state variables from which we `load()` the integrator state (`in_vars` and `out_vars` will generally point to the same array of pointers.)
out_vars	`double*`	Array of pointers to the state variables to which we `unload()` the integrator state (`in_vars` and `out_vars` will generally point to the same array of pointers.)
derivs_func	`DerivsFunc`	Function thats generates the function (the derivatives) to be integrated.
user_data	`void*`	A pointer to user defined data that will be passed to a DerivsFunc when called by the Integrator.

In addition to the above constructor, this class provides:

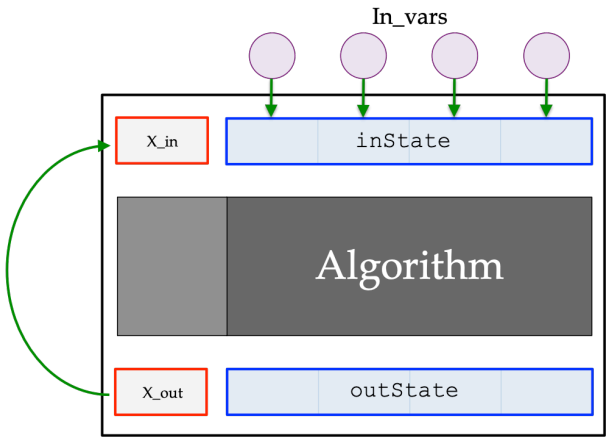
- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from class Integrator,
- and the following public member functions:

Public Member Functions

`void load()`

Overrides Integrator::load()

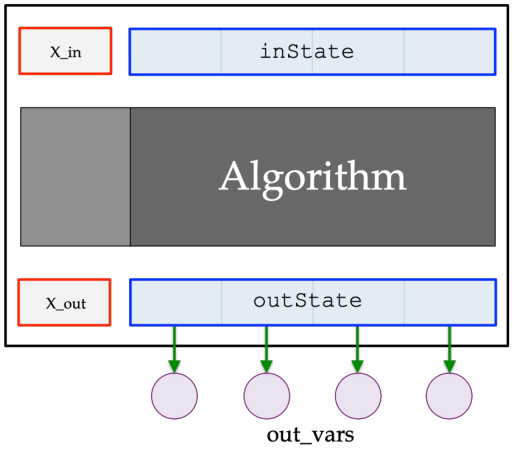
Load the integrator's initial state from the variables specified by **in_vars**. Set the initial value of the independent variable for the next step to the final value of the previous step.



`void unload()`

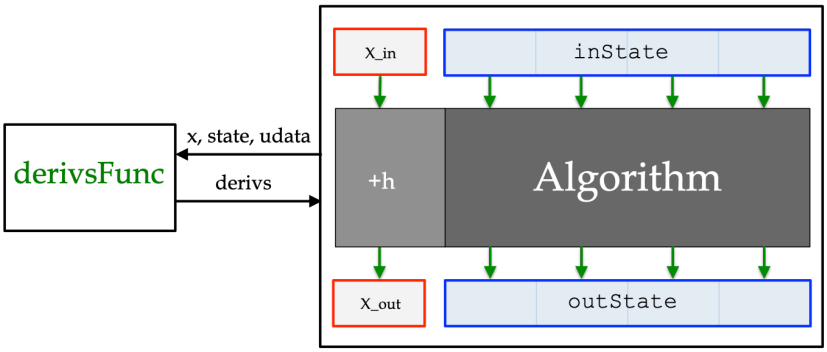
Overrides Integrator::unload()

Unload the integrator's result state to the variables specified by **out_vars**.



`virtual void step()`

Overrides Integrator::step()



Derived classes should override this method to calculate `outState` using some integration algorithm, using `X_in`, `inState`, and `derivs_func`, and `default_h`. The over-riding method should also pass the `user_data` when calling the `DerivsFunc`. The default algorithm is Euler.

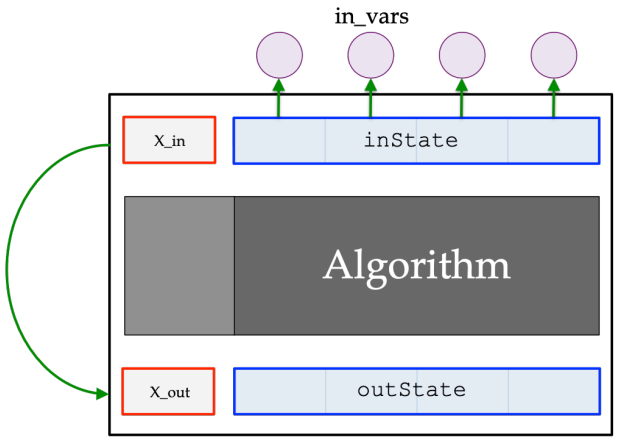
`void integrate()`

Inherited from Integrator::integrate()

`virtual void undo_integrate()`

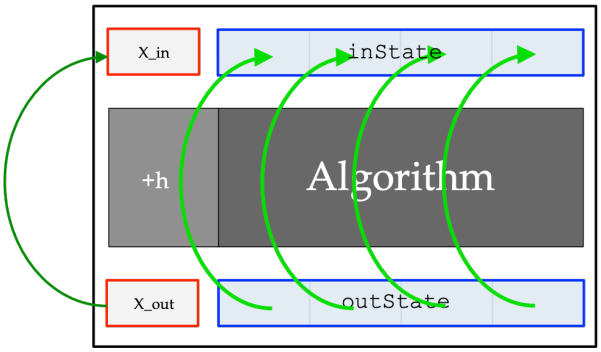
Overrides Integrator::undo_integrate()

Undo the effect of the last integration step.



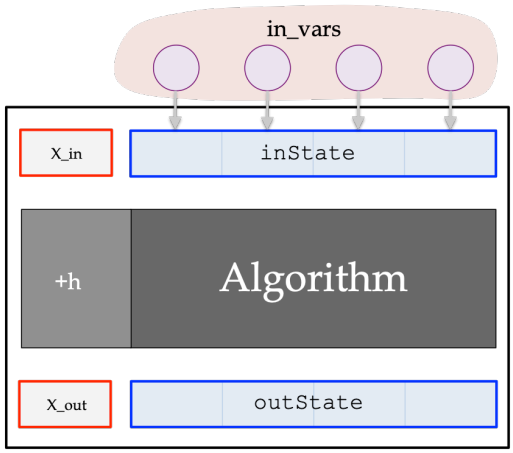
`void load_from_outState()`

Load `inState` from `outState`, rather than from the variables referenced by `in_vars`.



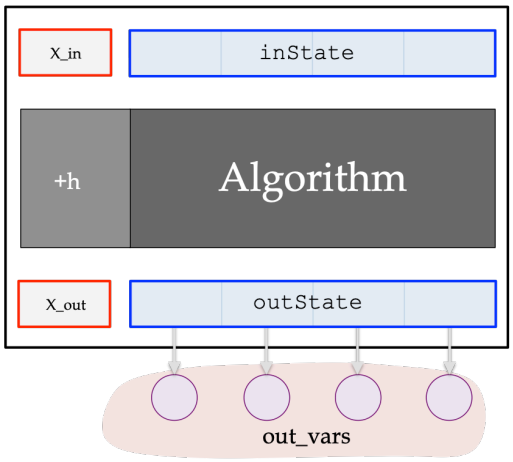
`double** set_in_vars( double* in_vars[])`

This function specifies the variables from which `inState` values are to be copied, when `load()` is called. The number of elements in this array must equal the number of state variables. Return a pointer to the previous array so that it can be deleted if necessary.



`double** set_out_vars( double* out_vars[])`

This function specifies the variables to which `outState` values are to be copied, when `unload()` is called. The number of elements in this array must equal the number of state variables. Return a pointer to the previous array so that it can be deleted if necessary.



`double getIndyVar()`

Inherited from Integrator::getIndyVar()

`double setIndyVar()`

Inherited from Integrator::setIndyVar()

Protected Member Functions

`advanceIndyVar()`

Inherited from Integrator::advanceIndyVar()

typedef RootErrorFunc

Description

This typedef defines a type of C/C++ function whose purpose is to specify the job of a `RootFinder` .

```
typedef double (*RootErrorFunc)( double x, double state[], RootFinder* root_finder, void* udata
```

where:

Parameter	Type	Direction	Description
x	`double`	In	Independent variable
state	`double*`	In	Array of state variable values
root_finder	`RootFinder*`	In	Class for finding the roots of a function.
udata	`void*`	In	A pointer to user_data.

A function of type **RootErrorFunc** should:

1. Specify a (math) function f(x) whose roots [x : f(x)=0] the RootFinder is meant to find.
    - f(x) is usually a function of the state variables. State variables are themselves functions of x.
2. Call `root_finder->find_roots(x, y)` , where y = f(x). If it returns 0.0, it's found a root of f(x).
3. Specify what to do as a result of finding a root.
4. Return the value returned by `root_finder->find_roots()` .

Example RootErrorFunc from the Cannonball example

```
double impact( double t, double state[], RootFinder* root_finder, void* udata) {
    double root_error = root_finder->find_roots(t, state[1]);
    if (root_error == 0.0) {
        root_finder->init();
        state[2] =  0.9 * state[2];
        state[3] = -0.9 * state[3];
    }
    return (root_error);
}
```

In this example :

- the independent variable is t.
- y = f(t) = state[1], that is the y (vertical) component of the cannonball position.
- When `root_finder->find_roots` returns 0.0, then the result of finding the root (i.e, [t:state[1]=0]) is to "bounce" the cannon ball, by negating the y component of the velocity, and reducing the magnitude of the velocity by 10%.

class FirstOrderODEVariableStepIntegrator

Derived from FirstOrderODEIntegrator.

Description

This class represents a first order ODE integrator whose step-size can be varied.

Data Members

Those inherited from FirstOrderODEIntegrator plus:

Member	Type	Access	Description
root_finder	`RootFinder*`	Private	Pointer to a RootFinder object.
root_error_func	`RootErrorFunc`	Private	Function that specifies what happens when a function-root is found.
last_h	`double`	Protected	Value of h used in the last integration step.

Constructor

```
FirstOrderODEVariableStepIntegrator( double h,
                                     unsigned int N,
                                     double* in_vars[],
                                     double* out_vars[],
                                     DerivsFunc dfunc,
                                     void* udata);
```

Constructor Parameters are those of FirstOrderODEIntegrator.

In addition to the above constructor, this class provides:

- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from FirstOrderODEIntegrator,
- and the following public member functions:

Public Member Functions

`void load()`

Inherited from FirstOrderODEIntegrator::load()

`void unload()`

Inherited from FirstOrderODEIntegrator::unload()

`void step()`

Overrides FirstOrderODEIntegrator::step()

This function calls the virtual function `variable_step()` (below) with the default step-size. Then, if a RootFinder has been specified using `add_Rootfinder()` (below), search that interval for roots .

`void integrate()`

Inherited from Integrator::integrate()

`double undo_integrate()`

Overrides FirstOrderODEIntegrator::undo_integrate()
Call `FirstOrderODEIntegrator::undo_integrate()` , and then return `last_h` .

`load_from_outState()`

Inherited from FirstOrderODEIntegrator::load_from_outState()

`set_in_vars()`

Inherited from FirstOrderODEIntegrator::set_in_vars()

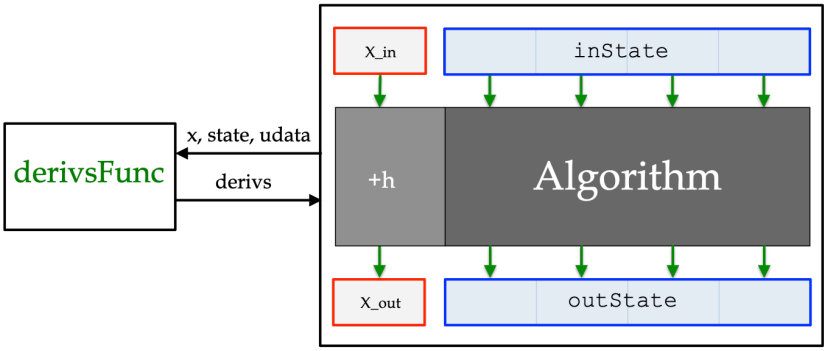
`set_out_vars()`

Inherited from FirstOrderODEIntegrator::set_out_vars()

`virtual void variable_step( double h)`

Parameter	Type	Description
h	`double`	Integration step-size that overrides the default step-size.

Derived classes should override this method to calculate `outState` using some integration algorithm, given `h`, `X_in`, `inState`, and `derivs_func`. The over-riding method should also pass the `user_data` when calling the `DerivsFunc`.



`void add_Rootfinder( RootFinder* root_finder, RootErrorFunc rfunc)`

Parameter	Type	Description
root_finder	`RootFinder*`	RootFinder object.
rfunc	`RootErrorFunc`	User supplied function whose purpose is to specify the job of a RootFinder.

Configure the integrator to find roots of state-element vs. independent-variable functions.

`double getIndyVar()`

Inherited from Integrator::getIndyVar()

`double setIndyVar()`

Inherited from Integrator::setIndyVar()

Protected Member Functions

`advanceIndyVar()`

Inherited from Integrator::advanceIndyVar()

class EulerIntegrator

Derived from FirstOrderODEVariableStepIntegrator.

Description

The Euler method is a first order numerical integration method. It is the simplest, explicit Runge-Kutta method.

Data Members

Those inherited from FirstOrderODEVariableStepIntegrator.

Constructor

```
EulerIntegrator( double h,
                 int N,
                 double* in_vars[],
                 double* out_vars[],
                 DerivsFunc func,
                 void* user_data)
```

Constructor Parameters are those of FirstOrderODEVariableStepIntegrator.

In addition to the above constructor, this class provides:

- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from FirstOrderODEVariableStepIntegrator,
- and the following public member functions:

Public Member Functions

- All of the Public Member Functions of FirstOrderODEVariableStepIntegrator, plus :

`void variable_step( double h)`

Overrides FirstOrderODEVariableStepIntegrator::variable_step()

Calculates `outState` from `h`, `X_in`, `inState`, and `derivs_func`, using the Euler method.

class HeunsMethod

Derived from FirstOrderODEVariableStepIntegrator.

Description

This integrator implements Heun's Method.

Data Members

Those inherited from FirstOrderODEVariableStepIntegrator.

Constructor

```
HeunsMethod( double h,
             int N,
             double* in_vars[],
             double* out_vars[],
             DerivsFunc func,
             void* user_data)
```

Constructor Parameters are those of FirstOrderODEIntegrator.

In addition to the above constructor, this class provides:

- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from FirstOrderODEVariableStepIntegrator,
- and the following public member functions:

Public Member Functions

- All of the Public Member Functions of FirstOrderODEVariableStepIntegrator.

`void variable_step( double h)`

Overrides FirstOrderODEVariableStepIntegrator::variable_step()

Calculates `outState` from `h`, `X_in`, `inState`, and `derivs_func`, using the Heun's method.

class RK2Integrator

Derived from FirstOrderODEVariableStepIntegrator.

Description

`RK2Integrator` implements the second order, explicit, Runge-Kutta method whose Butcher tableau is as follows.

0		
1/2	1/2	
	0	1

Data Members

Those inherited from FirstOrderODEVariableStepIntegrator.

Constructor

```
RK2Integrator( double h,
               int N,
               double* in_vars[],
               double* out_vars[],
               DerivsFunc func,
               void* user_data)
```

Constructor Parameters are those of FirstOrderODEIntegrator.

In addition to the above constructor, this class provides:

- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from FirstOrderODEVariableStepIntegrator,
- and the following public member functions:

Public Member Functions

- All of the Public Member Functions of FirstOrderODEVariableStepIntegrator.

`void variable_step( double h)`

Overrides FirstOrderODEVariableStepIntegrator::variable_step()

Calculates `outState` from `h`, `X_in`, `inState`, and `derivs_func`, using the Runge-Kutta 2 method.

class RK4Integrator

Derived from FirstOrderODEVariableStepIntegrator.

Description

`RK4Integrator` implements the fourth order, explicit, Runge-Kutta method whose Butcher tableau is as follows.

0				
1/2	1/2			
1/2	0	1/2		
1	0	0	1	
	1/6	1/3	1/3	1/6

Data Members

Those inherited from FirstOrderODEVariableStepIntegrator.

Constructor

```
RK4Integrator( double h,
               int N,
               double* in_vars[],
               double* out_vars[],
               DerivsFunc func,
               void* user_data)
```

Constructor Parameters are those of FirstOrderODEIntegrator.

In addition to the above constructor, this class provides:

- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from FirstOrderODEVariableStepIntegrator,
- and the following public member functions:

Public Member Functions

- All of the Public Member Functions of FirstOrderODEVariableStepIntegrator.

`void variable_step( double h)`

Overrides FirstOrderODEVariableStepIntegrator::variable_step()

Calculates `outState` from `h`, `X_in`, `inState`, and `derivs_func`, using the Runge-Kutta 4 method.

class RK3_8Integrator

Derived from FirstOrderODEVariableStepIntegrator.

Description

`RK3_8Integrator` implements the fourth order, explicit, Runge-Kutta method whose Butcher tableau is as follows.

0				
1/3	1/3			
2/3	-1/3	1		
1	1	-1	1	
	1/8	3/8	3/8	1/8

Data Members

Those inherited from FirstOrderODEVariableStepIntegrator.

Constructor

```
RK3_8Integrator( double h,
                 int N,
                 double* in_vars[],
                 double* out_vars[],
                 DerivsFunc func,
                 void* user_data)
```

Constructor Parameters are those of FirstOrderODEIntegrator.

In addition to the above constructor, this class provides:

- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from FirstOrderODEVariableStepIntegrator,
- and the following public member functions:

Public Member Functions

- All of the Public Member Functions of FirstOrderODEVariableStepIntegrator.

`void variable_step( double h)`

Overrides FirstOrderODEVariableStepIntegrator::variable_step()

Calculates `outState` from `h`, `X_in`, `inState`, and `derivs_func`, using the Runge-Kutta 3/8 method.

class ABM2Integrator

Derived from FirstOrderODEIntegrator.

Description

The ABM2Integrator implements the second-order Adams-Bashforth-Moulton predictor/corrector method. Adams methods maintain a history of derivatives rather than calculating intermediate values like Runge-Kutta methods.

Data Members

Those inherited from FirstOrderODEIntegrator.

Constructor

```
ABM2Integrator ( double h,
                 int N,
                 double* in_vars[],
                 double* out_vars[],
                 DerivsFunc func,
                 void* user_data)
```

Constructor Parameters are those of FirstOrderODEIntegrator.

In addition to the above constructor, this class provides:

- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from [FirstOrderODEIntegrator.

class ABM4Integrator

Derived from FirstOrderODEIntegrator.

Description

The ABM2Integrator implements the second-order Adams-Bashforth-Moulton predictor/corrector method. Adams methods maintain a history of derivatives rather than calculating intermediate values like Runge-Kutta methods.

Data Members

Those inherited from FirstOrderODEIntegrator.

Constructor

```
ABM4Integrator ( double h,
                 int N,
                 double* in_vars[],
                 double* out_vars[],
                 DerivsFunc func,
                 void* user_data)
```

Constructor Parameters are those of FirstOrderODEIntegrator.

In addition to the above constructor, this class provides:

- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from FirstOrderODEIntegrator.

typedef Derivs2Func

Description

This typedef defines a type of C/C++ function whose purpose is to populate an array of accelerations, given velocities and positions.

```
typedef void (*Derivs2Func)( double t, double x[], double v[], double a[], void* udata);
```

where:

Parameter	Type	Direction	Description
t	`double`	IN	Independent variable.
x	`double*`	IN	Array of position values.
v	`double*`	IN	Array of velocity values.
a	`double*`	OUT	Array into which accelerations are to be returned.
udata	`void*`	IN	Pointer to user_data.

Example

```
void G( double t, double x[], double v[], double g_out[], void* udata) {
    MassSpringDamper* msd = (MassSpringDamper*)udata;
    g_out[0] = -(msd->k/msd->mass) * x[0]
               -(msd->c/msd->mass) * v[0];
}
```

class EulerCromerIntegrator

Derived from Integrator.

Description

EulerCromer is integration method that conserves energy in oscillatory systems better than Runge-Kutta. So, it's good for mass-spring-damper systems, and orbital systems.

It calculates the next state, from the current state as follows:

$$v_{n+1} = v_n + a(v_n, x_n, t)\Delta t$$
$$x_{n+1} = x_n + v_{n+1}\Delta t$$

a(v(n), x(n), t) [above] is the function of type `Derivs2Func` below.

Data Members

Those inherited from Integrator plus:

Member	Type	Access	Description
nDimensions	`unsigned int`	Protected	Number of dimensions in position, velocity, and acceleration vectors. Typically 1,2, or 3.
pos_p	`double**`	Protected	Array of pointers to variables from which we `load()` and to which we `unload()` the position values .
vel_p	`double**`	Protected	Array of pointers to variables from which we `load()` and to which we `unload()` the velocity values .
pos_in	`double*`	Protected	Position input array.
vel_in	`double*`	Protected	Velocity input array.
pos_out	`double*`	Protected	Position output array.
vel_out	`double*`	Protected	Velocity output array.
g_out	`double*`	Protected	Array of accelerations returned from gderivs.
gderivs	`Derivs2Func`	Protected	A function that returns accelerations.
last_h	`double`	Value of h used in the last integration step.	

Constructor

```
EulerCromerIntegrator(double dt,
                      int N,
                      double* xp[],
                      double* vp[],
                      Derivs2Func gfunc,
                      void* user_data)
```

Parameter	Type	Description
dt	`double`	Default time step value. Sets Integrator::default_h.
N	`int`	Sets nDimensions above.
xp	`double*`	Sets pos_p above.
vp	`double*`	Sets vel_p above.
gfunc	`Derivs2Func`	Sets gderivs above.
user_data	`void*`	Sets Integrator::user_data.

In addition to the above constructor, this class provides:

- a copy constructor,
- a destructor,
- an assignment operator,
- an insertion operator,
- the public member functions inherited from Integrator.

Public Member Functions

`void step( double dt)`

Parameter	Type	Description
dt	`double`	Integration step-size that overrides the default step-size.

This function calculates `pos_out` and `vel_out` from `dt`, `X_in`, `pos_in`, `vel_in`, `f_func`, and `gfunc` using the Euler-Cromer method.

`void step()`

This function calls `step(dt)` (above) with the default step-size.

`void load()`

Overrides Integrator::integrate() Load the integrator's initial state from the variables specified by **xp**, and **vp**. Set the initial value of the independent variable for the next step to the final value of the previous step.

`void unload()`

Overrides Integrator::integrate()

Unload the integrator's result state (**pos_out**, and **vel_out**) to the variables specified by **xp**, and **vp**.

`void integrate()`

Inherited from Integrator::integrate()

`double undo_integrate()`

Overrides Integrator::undo_integrate()

Undo the effect of the last integration step.

`double getIndyVar()`

Inherited from Integrator::getIndyVar()

`double setIndyVar()`

Inherited from Integrator::setIndyVar()

Protected Member Functions

`advanceIndyVar()`

Inherited from Integrator::advanceIndyVar()

enum SlopeConstraint

Description

Value	Meaning
Negative	Require slope of the function to be negative at the root.
Unconstrained	No slope constraint.
Positive	Require slope of the function to be positive at the root.

class RootFinder

Description

The RootFinder class uses the Regula-Falsi method to find roots of a math function. A root is a value of **x** such that **f(x)=0**.

Data Members

Member	Type	Access	Description
f_upper	`double`	Private	Error-function value upper bound.
x_upper	`double`	Private	Independent variable value upper bound.
upper_set	`bool`	Private	True = bound is valid. False = not valid.
f_lower	`double`	Private	Error-function value lower bound.
x_lower	`double`	Private	Independent variable value lower bound.
lower_set	`bool`	Private	True = bound is valid. False = not valid.
prev_f_error	`double`	Private	Absolute value of the previous root function value.
f_error_tol	`double`	Private	How close is close enough.
iterations	`int`	Private	Number of Regula Falsi iterations.
slope_constraint	`SlopeConstraint`	Private	Find roots with this slope sign.
f_slope	`SlopeConstraint`	Private	Current root function slope.

Constructors

`RootFinder()`

Default constructor that calls `void RootFinder::init()` below.

`RootFinder(double tolerance, SlopeConstraint constraint)`

Parameter	Type	Description
tolerance	`double`	Error tolerance.
constraint	`SlopeConstraint`	

Public Member Functions

`void init( double tolerance, SlopeConstraint constraint)`

Initialize the RootFinder with the given tolerance, and SlopeConstraint.

`void RootFinder::init()`

Initialize the RootFinder with the method above with:

- tolerance = `0.00000000001`
- slope_constraint = `Unconstrained`

`double find_roots( double x, double f_error )`

- Returns **DBL_MAX** if no root is detected.
- Returns **0.0** if a root is detected, and the estimated error in f(x) is within tolerance.
- Returns **an estimated correction in x** if a root is detected, but the estimated error in f(x) is not within tolerance.