Aircraft Sim Design

Project: Aircraft Simulation

Using Newton's Second Law, and only the forces of engine thrust, and aerodynamic drag, described in the following pages, complete a Trick based simulation which continuously computes the position and velocity of an aircraft.

Simplifications

In this simulation, we'll assume that the aircraft always points in the direction of flight. In the aircraft body coordinates, thrust is directed in the +x direction and drag in the –x direction.

The aircraft will only move in two-dimensions, like on a map. We will not consider the forces of lift, or gravity, which would require that our simulation be three-dimensional.

Control

In this simulation, we'll control the aircraft by specifying:

1) a desired speed
2) a desired heading.

General Strategy for Developing a Dynamic Simulation

"Dynamic" means "characterized by change".

In a dynamic simulation things like position, and velocity, continuously change over time. Lots of other things might change as well depending on what you are simulating. The amount that something changes per unit of time is its **rate of change,** or **time derivative.**

For example:

- Acceleration is a rate of velocity change .
- Velocity is a rate of position change.
- Current is a rate of change of charge.

If we know the rate of change of something, then we can add up all the little changes over time to keep track of its value. This "adding up the little pieces" is called "numerical integration".

This is the basic strategy for designing Trick dynamic simulations. We calculate rates, and then we numerically integrate them.

General Strategy for Motion

To determine the motion of an object we generally start with Newton's Second Law:

$$F = ma$$

That is, force equals mass times acceleration.

This allows us to determine the rates that effect motion, that is: acceleration and velocity.

Solving for acceleration, we get :

$$a = F/m$$

This is the form of Newton's Law that we generally use.

So, if we have a force acting on a mass, we can determine its acceleration. Then, we can then numerically integrate that acceleration to get velocity and then integrate the velocity to get position.

General Design of Our Simulation

In our simulation our rates will be acceleration, velocity. For each time step (δt) of our simulation, we'll integrate our rates over the time period t → t + δt to get the next velocity, position.

A Trick integration job will calculate this for us.

Acceleration

To calculate acceleration, we'll use Newton's Second Law as usual. The force is the total force acting on the aircraft.

(1) $$\vec{a} = \frac{\vec{F_{total}}}{m}$$

The total force on our aircraft is the sum of:

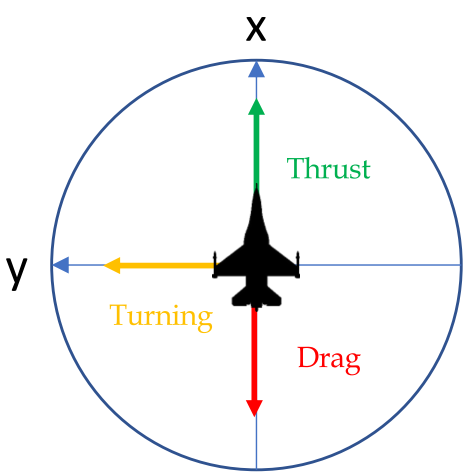
- the thrust force produced by the aircraft's engine and
- the drag force produced by the aircraft pushing through the atmosphere.

(2) $$\vec{F_{total}} = \vec{F_{thrust}} + \vec{F_{drag}} + \vec{F_{turning}}$$

Notice that force is a vector. That is, it has a <u>magnitude</u>, and a <u>direction</u>.

If we affix a "Body" coordinate system to the aircraft as shown, then the thrust force will act in the +x direction, and the drag force in the –x direction. That is, thrust and drag that point in opposite directions.

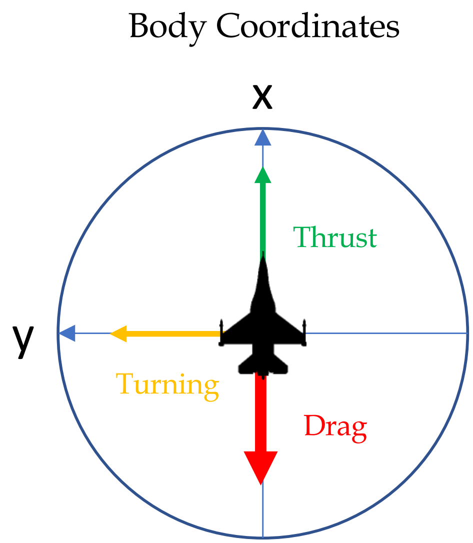
Body Coordinates



Drag Force

The magnitude of the drag force is proportional to the square of the aircraft's speed. We'll call our constant of proportionality "K_{drag}".

(3) $$\vec{F}_{drag_{body}} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}_{body} \cdot K_{drag} \cdot speed^2$$



By "speed", we mean the magnitude of the velocity :

(4) $$speed = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

The direction of the drag force is opposite the direction of velocity. So, the drag force is:

Note: Column vectors in body coordinates are of the form: $\begin{bmatrix} x \\ y \end{bmatrix}_{body}$

Column vectors in world coordinates are of the form: $\begin{bmatrix} n \\ w \end{bmatrix}_{world}$

Thrust Force

The magnitude of the thrust force produced by our engine can be anywhere between 0 and some value that we specify as the maximum ($thrust_{MAX}$). For our aircraft.

We said in the beginning that we want to control our aircraft by setting a "desired speed". The amount of thrust produced obviously determines the speed.

So, how much thrust do we need to reach and maintain a desired speed?

- To increase speed, we must accelerate, in which case magnitude of the thrust must be greater than that of the drag.
- **To maintain the same speed acceleration must be zero, in which case the thrust and drag magnitudes must be equal.**
- To decrease speed, we must decelerate, in which case thrust magnitude must be less than that of drag.

Therefore, the thrust we require to reach and maintain a desired speed must be the same magnitude as the drag force at that desired speed.

(5) $$thrust_{desired} = K_{drag} \times speed^2_{desired}$$

So, to go our desired speed, this is the magnitude of thrust we need.

Thrust Force

If our engine can produce the desired thrust, then we can set the throttle to produce that thrust ($thrust_{actual}$) and we'll speed up to that speed. But what if we can't? The most thrust we can produce is $thrust_{MAX}$., when we're at full throttle.

So, we must limit $thrust_{actual}$ to that which our engine can actually produce :

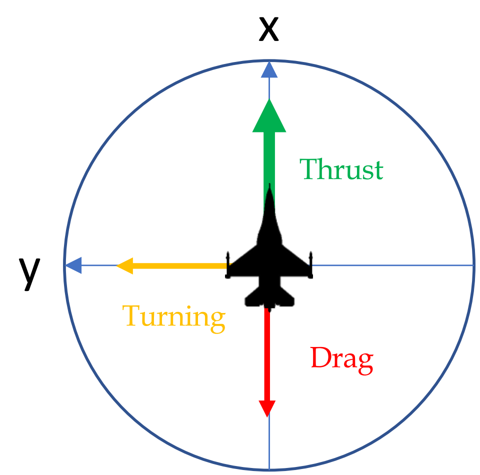
(6) $$thrust_{actual} = \begin{cases} thrust_{MAX}, & \text{if } thrust_{desired} > thrust_{MAX} \\ 0, & \text{if } thrust_{desired} < 0 \\ thrust_{desired}, & \text{otherwise} \end{cases}$$

So, we imagine that we set our throttle to produce $thrust_{actual}$.

Thrust Force

(7) $$\vec{F}_{thrust_{body}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{body} \cdot thrust_{actual}$$

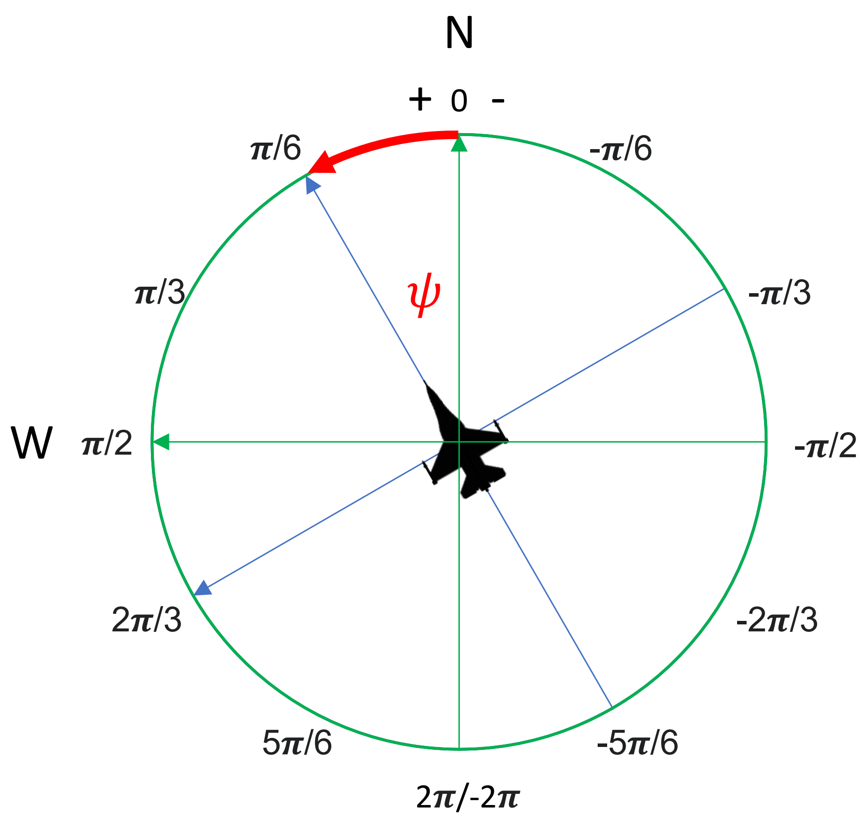
Direction magnitude



So, the magnitude of the thrust force vector is $thrust_{actual}$, and the direction is [1,0] in Body coordinates.

Aircraft Heading

The direction that an aircraft is moving is it's heading (ψ). See: "Calculating Heading from Velocity" below.



If the aircraft is moving directly north, ψ = 0. Directly west is $\boldsymbol{\pi}/2$. Directly east is -$\boldsymbol{\pi}/2$.

A heading rate is a change in heading per change in time.

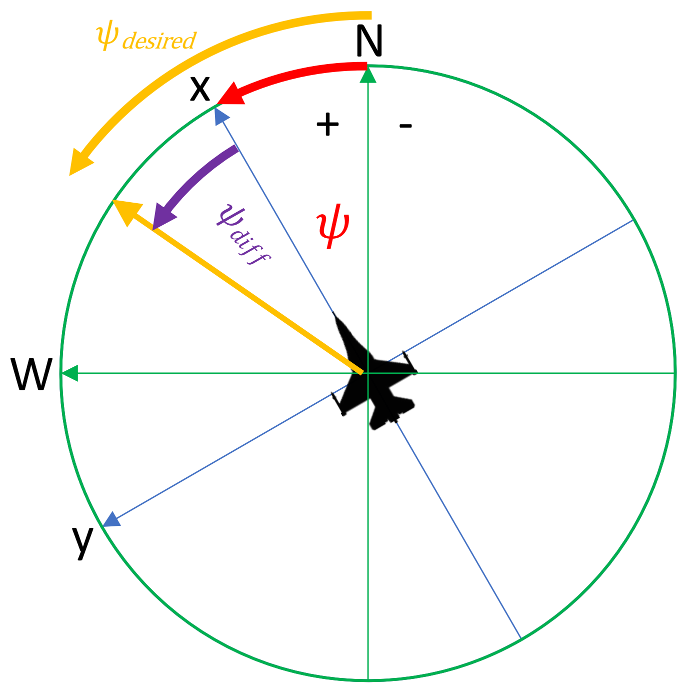
$$\dot{\psi} = \Delta\psi / \Delta t$$

For example, if you were spinning in an office chair at $\boldsymbol{\pi}/2$ radians per second, then it would take 4 seconds to make one full counter-clockwise turn. (There's 2$\boldsymbol{\pi}$ radians in a circle).

If you were spinning in an office chair at -$\boldsymbol{\pi}/6$ radians per second, then it would take 12 seconds to make one full clockwise turn.

Aircraft Heading

To control our heading, we'd like to calculate a heading rate that is proportional to the difference between our current heading and our desired heading.



For the sake of discussion, let's define:

(8) $$\psi_{diff} = \psi_{desired} - \psi$$

Notice (in the figure) that ψ_{diff} is in the direction that we want to turn. But, what if ($|\ \psi_{diff}\ | > \pi$)? That is, it's greater than 180 degrees?

Do you ever have to turn more than π radians (180 degrees) to face any direction? No, you don't.

Instead you make the smaller turn in opposite direction.

Aircraft Heading

Let's now define ψ_{error} as the actual direction of turning.

$$\text{(9)} \quad \psi_{error} = \begin{cases} \psi_{diff} - 2\pi\ , & \text{if } \psi_{diff} > \pi \\ \psi_{diff} + 2\pi\ , & \text{if } \psi_{diff} < -\pi \\ \psi_{diff}\ , & \text{otherwise} \end{cases}$$

This is how we will calculate our desired heading rate:

$$\text{(10)} \qquad \dot{\psi}_{desired} = G_{\psi} \cdot \psi_{error}$$

That is, our desired heading rate is proportional to the amount and direction that we need to turn.

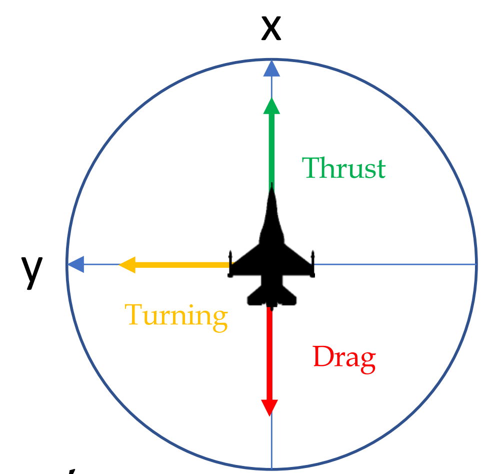
G_{ψ} is a proportionality constant.

In your simulation use $G_{\psi} = 0.1$.

Turning Force

The turning force we needed to turn the desired rate is :

$$\vec{F}_{turning_{body}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{body} \cdot speed \cdot \dot{\psi}_{desired} \cdot m \tag{11}$$



Like our desired speed, just we want something doesn't mean that the aircraft is capable of it. In our case we need to limit the magnitude of our force to 1 G (force of gravity), that is 9.8 m/s^2 * m. So if our mass is 5000 kg, we need to limit our turning force to between -49000 .. 49000 Newtons.

Total Force

But we need that total force in world coordinates so we can move our aircraft around in the world.

So, if our heading of our aircraft is ψ, what is our direction vector expressed in world coordinates?

(12) $$\vec{F_{total_{world}}} = \begin{bmatrix} cos\psi & -sin\psi \\ sin\psi & cos\psi \end{bmatrix} \vec{F_{total_{body}}}$$

Knowledge Check: What does this equation mean? What's another way to express this?

Calculating Heading from Velocity

```
double northWestToPsi (double (&p)[2]) {
    double psi;
    double mag = sqrt( p[0]*p[0] + p[1]*p[1] );
    if (mag > 0.0) {
        psi = atan2( p[1], p[0] );

    } else {
        std::cerr << "Error: Arguments to northWestToPsi() are zero."
                  << std::endl;
        psi = 0.0;
    }
    return psi;
}
```

To calculate psi from velocity:

```
heading = northWestToPsi(vel);
```

All the Equations We Need for Our Simulation

1 $$\vec{a} = \frac{\vec{F_{total}}}{m}$$

2 $$\vec{F_{total}} = \vec{F_{thrust}} + \vec{F_{drag}} + \vec{F_{turning}}$$

3 $$\vec{F_{drag_{body}}} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}_{body} \cdot K_{drag} \cdot speed^2$$

4 $$speed = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

5 $$thrust_{desired} = K_{drag} \times speed_{desired}^2$$

6 $$thrust_{actual} = \begin{cases} thrust_{MAX}, & \text{if } thrust_{desired} > thrust_{MAX} \\ 0, & \text{if } thrust_{desired} < 0 \\ thrust_{desired}, & \text{otherwise} \end{cases}$$

7 $$\vec{F_{thrust_{body}}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{body} \cdot thrust_{actual}$$

(8) $$\psi_{diff} = \psi_{desired} - \psi$$

(9) $$\psi_{error} = \begin{cases} \psi_{diff} - 2\pi, & \text{if } \psi_{diff} > \pi \\ \psi_{diff} + 2\pi, & \text{if } \psi_{diff} < -\pi \\ \psi_{diff}, & \text{otherwise} \end{cases}$$

(10) $$\dot{\psi}_{desired} = G_{\psi} \cdot \psi_{error}$$

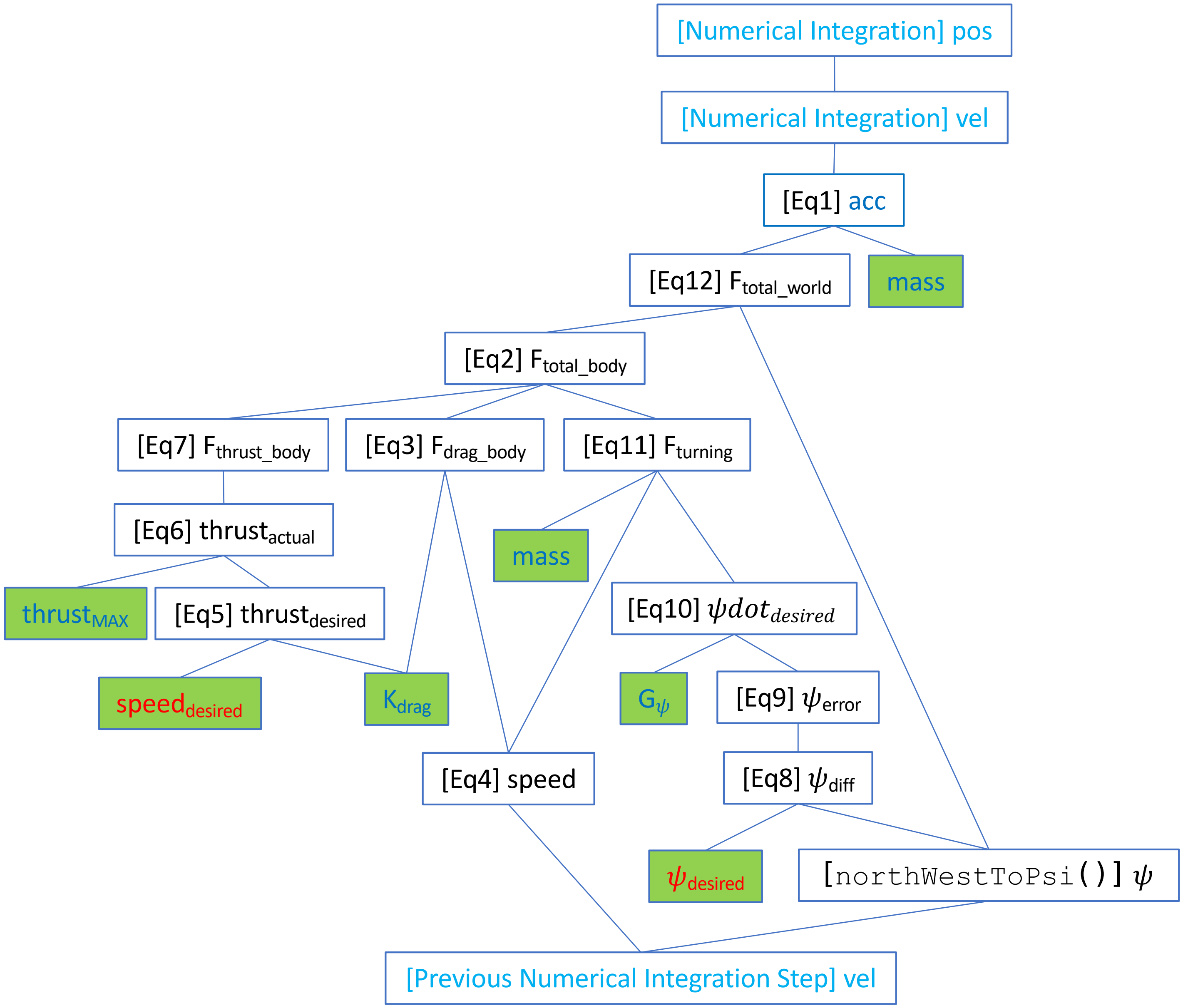
(11) $$\vec{F_{turning_{body}}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{body} \cdot speed \cdot \dot{\psi}_{desired} \cdot m$$

(12) $$\vec{F_{total_{world}}} = \begin{bmatrix} cos\psi & -sin\psi \\ sin\psi & cos\psi \end{bmatrix} \vec{F_{total_{body}}}$$

Default Values

pos = <0.0, 0.0>	meters
vel = <100.0, 0.00>	meters/second
mass = 5000.0	kg
thrust_max = 45000.0	Newtons
K_drag = 0.72	
heading_rate_gain = 0.1	
desired_heading = $\pi/4$	radians
desired_speed = 200.0	meters/second

Computational Dependencies



What insights can we gain from this diagram to help us organize our code?

Have Fun!